Complex Analysis

Midterm Exam

7 December 2020

The exam consists of five problems worth a total of nine points; you get one initial bonus point. This file only contains Problem 1. Your solution to this problem should consist of a single PDF file (named `problem1.pdf`). All files containing your solutions need to be uploaded together, under the Midterm Exam assignment on Nestor, **before 11:20** (UTC+1).

Problem 1 [1.5 points]

Let $f\colon \mathbb{C} \to \mathbb{C}$ be the function defined by

$$f(z + \mathbb{i}y) = \begin{cases} \dfrac{\sqrt{x^2y^4} + \mathbb{i}\sqrt{x^4y^2}}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{otherwise.} \end{cases}$$

Check whether the Cauchy–Riemann equations hold for f at the origin and decide whether f is differentiable at the origin.

[**Note:** You need only consider the partial derivatives at one point.]

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Problem 2 [2 points]

a) Let $z = re^{\mathbb{i}\theta}$ and let

$$f(z) = u(r,\theta) + \mathbb{i}\, v(r,\theta)$$

be a function that is differentiable at $z_0 = r_0 e^{\mathbb{i}\theta_0}$. Show that the following equalities hold:

$$r_0 \frac{\partial u}{\partial r}(r_0,\theta_0) = \frac{\partial v}{\partial \theta}(r_0,\theta_0),$$
$$\frac{\partial u}{\partial \theta}(r_0,\theta_0) = -r_0 \frac{\partial v}{\partial r}(r_0,\theta_0).$$

[**Hint:** Recall the chain rule for real multivariate functions.]

b) Suppose that

$$v = \frac{\cos\theta + \sin\theta}{r}$$

is the imaginary part of a function $f(z) = f(re^{\mathbb{i}\theta})$ that is analytic on the domain $\mathbb{C}^\times = \mathbb{C} \setminus \{0\}$ and satisfies $f(-1) = 1 - \mathbb{i}$. Determine $f(z)$, i.e. express f as a function of z.

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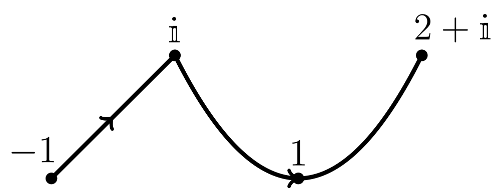
Problem 3 [2 points]

Let Γ be the contour depicted below, consisting of:

i) the line segment connecting -1 and $\mathbb{i}$,

ii) the parabola segment connecting $\mathbb{i}$ and $2+\mathbb{i}$ and passing through 1.



Determine the following integrals:

a) $\displaystyle\int_\Gamma \bar{z}\,\mathrm{d}z$ b) $\displaystyle\int_\Gamma z\,\mathrm{d}z$

Write your answer in the form $x + \mathbb{i}y$ with $x, y \in \mathbb{R}$.

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university of groningen

The exam consists of five problems worth a total of nine points; you get one initial bonus point. This file only contains Problem 4. Your solution to this problem should consist of a single PDF file (named `problem4.pdf`). All files containing your solutions need to be uploaded together, under the MIDTERM EXAM assignment on Nestor, **before 11:20** (UTC+1).

Problem 4 [2 points]

Let Γ be the circle $|z - \mathbb{i}| = 1$. Determine the integral

$$\int_{\Gamma} \frac{z \sin(\pi z)}{(z^2+1)^2} \, \mathrm{d}z.$$

Write your answer in the form $x + \mathbb{i}y$ with $x, y \in \mathbb{R}$.

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The exam consists of five problems worth a total of nine points; you get one initial bonus point. This file only contains Problem 5. Your solution to this problem should consist of a single PDF file (named `problem5.pdf`). All files containing your solutions need to be uploaded together, under the MIDTERM EXAM assignment on Nestor, **before 11:20** (UTC+1).

Problem 5 [1.5 points]

Let D be a bounded domain and let $\overline{D} = D \cup \partial D$ (the union of D and its boundary). Let $f\colon \overline{D} \to \mathbb{C}$ be a non-constant continuous function that is analytic on D. Suppose that there exists a point $z^* \in D$ such that

$$|f(z^*)| < \min_{z \in \partial D} |f(z)|.$$

Show that f has a zero in D.